# University of Freiburg – Mathematical Institute Winter term 2024/25

Supplements of the module handbooks

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# Study Planning

Dear Students of Mathematics,

The course catalogue provides information about the course offers of the Mathematical Institute for the current semester. Information on the course of study, which courses you can and which courses you have to pick, can be found on the information pages for each programme at https://www.math.uni-freiburg.de/nlehre/. Please note that there may be different versions of examination regulations for a study programme.

You are welcome to use the advisory services of the Mathematical Institute if needed: study counseling by the programme coordinator, study counseling by the individual departments, and counseling by lecturers (office hours are listed on the personal webpages linked in the Institute's directory).

Please note:

- The two **Bachelor programmes** as well as the **Master of Education as additional subject** programmes begin with the basic lectures Analysis I and II and Linear Algebra I and II, on which most other mathematics courses are based on. In case you can only start with one of the two basic lectures due to the combination of subjects in the two-major Bachelor programme, you can find variants for the course of study on the programme's information page.
- As an orientation requirement, the two exams for Analysis I and Linear Algebra I must be passed by the end of the 3rd semester in the **B.Sc. programme**, and at least one of the two in the **two-major Bachelor programme**.
- There are no further regulations regarding the structure of your individual course of study and no formal prerequisites for attending courses (except for the limited number of places in each seminar or proseminar). However, you must ensure that you have the necessary prior knowledge independently.
- In the **M.Sc. Mathematics programme**, please note that you may take a maximum of two of the four oral exams with the same examiner.
- To what extent the material of advanced lectures is sufficient as **preparation for final theses and exams** or should be supplemented must be agreed upon soon enough with the supervisor of the thesis or the examiners. This applies in particular to the oral exam in the specialization module of the M.Sc. programme.

#### Language

Courses with the abbreviation "D" are offered in German, courses with the abbreviation "E" in English. Exercises for English lectures can often also be completed in German, and vice versa.

Presentations in seminars can usually be given in German and English; the abbreviation "D/E" indicates this possibility.

# Usability of Courses

For each course, the "Usability" section indicates in which modules from which degree programmes it can be used. For the degree programmes, the following abbreviations are used:

2 H f B 2 1	Two-Major Bachelor Programme
BSc21	Bachelor of Science in Mathematics, regulations of 2021
BScInfo19	Bachelor of Science in Computer Science, regulations of 2019
BScPhys22	Bachelor of Science in Physics, regulations of of 2022
MEd18	Master of Education in Mathematics
MEdual24	Master of Education – "dual study programme"
MEH21	Master of Education, Mathematics as an additional subject with 120 ECTS points
MEB21	Master of Education, Mathematics as an additional subject with 90 ECTS points
MSc14	Master of Science in Mathematics
MScData24	Master of Science in Mathematics in Data and Technology

As a general rule, no courses that were already used in the underlying Bachelor programme may be completed in a Master programme. If you have any questions, please contact the programme coordination.

Please note:

• It is allowed to use higher-level lectures, typically offered for the M.Sc. Mathematics programme, for electives in other study programmes; however, due to the required prior knowledge, they will only be suitable in exceptional cases. If a course can be used for a module, does not necessarily mean that it is suitable for a module. Conversely,

extreme cases are not listed though possible (e.g. a lecture such as "Differential Geometry II" as a specialisation module in the M.Ed. study proramme).

• In the B.Sc. Mathematics according to the regulations of 2021, in addition to the compulsory area, at least three 4-hour lectures with 2-hour exercises (9 ECTS points each) must be completed. At least one of these must be from the field of pure mathematics. You can see which of the lectures count as pure mathematics by checking whether it is approved for the module "Pure Mathematics" in the M.Sc. Mathematics programme.

# Examination Requirements and Pass/Fail Assessments

The section "Usability" will be supplemented at the beginning of the lecture period with information on which graded examinations and pass/fail assessments are required for its use in the respective module or study area. This information supplements the module handbooks in terms of examination and accreditation law and have been approved by the Mathematics Study Commission on October 31st, 2024.

Please note:

- Deviations from the specified type of graded examination are permitted if, due to circumstances beyond the examiner's control, the intended type of examination is not suitable or would involve disproportionate effort. The same applies to pass/fail assessments.
- If a course is approved as an elective module in a non-listed study programme, the requirements follow
  - the elective module of the B.Sc. programme, if examination achievements are required,
  - the elective module of the M.Sc. programme, if only pass/fail assessments are required.

If the corresponding modules are not offered, please contact the programme coordination of the Mathematical Institute.

- If written exercise assignments are required as a study achievement, these are usually weekly exercise assignments, or bi-weekly in the case of a one-hour exercise. Depending on the start, end, rhythm, and individual breaks, there can be between 5 and 14 exercise sheets. The number of points achievable per exercise sheet can vary.
- For practical exercises, this applies analogously to programming assignments.

# **Research Areas for Final Theses**

Information on Bachelor and Master theses in Mathematics can be found here:

https://www.math.uni-freiburg.de/nlehre/en/studiendekanat/faq/abschlussarbeit

The following list gives you an overview of the fields from which the professors and lecturers of the Mathematical Institute typically assign topics for final theses. If you are interested in a thesis, please arrange an appointment early!

Prof.Dr. Sören Bartels	Applied Mathematics, Partial Differential Equations, and Numerics
Prof.Dr. Harald Binder	Medical Biometrics and Applied Statistics
JProf. Dr. David Criens	Stochastic Analysis, Probability Theory, and Financial Mathematics
Prof. Dr. Moritz Diehl	Numerics, Optimization, Optimal Control
Prof. Dr. Patrick W. Dondl	Applied Mathematics, Calculus of Variations, Partial Differential Equations, and Numerics
Prof. Dr. Sebastian Goette	Differential Geometry, Topology, and Global Analysis
Prof. Dr. Nadine Große	Differential Geometry and Global Analysis
Prof.Dr. Annette Huber-Klawitter	Algebraic Geometry and Number Theory
PDDr. Markus Junker	Mathematical Logic, Model Theory
Prof. Dr. Stefan Kebekus	Algebra, Complex Analysis, Complex and Algebraic Geometry
Prof. Dr. Ernst Kuwert	Partial Differential Equations, Calculus of Variations
Prof. Dr. Eva Lütkebohmert-Holtz	Financial Mathematics, Risk Management and Regulation
Prof. Dr. Amador Martín Pizarro	Mathematical Logic, especially Model Theory
Prof. Dr. Heike Mildenberger	Mathematical Logic, especially Set Theory and Infinite Combinatorics
JProf. Dr. Abhishek Oswal	Algebra
Prof. Dr. Peter Pfaffelhuber	Stochastics, Biomathematics
Prof. Dr. Angelika Rohde	Mathematical Statistics, Probability Theory
Prof. Dr. Michael Růžička	Applied Mathematics and Partial Differential Equations
JProf. Dr. Diyora Salimova	Applied Mathematics, Partial Differential Equations, Machine Learning, and Numerics
Prof. Dr. Thorsten Schmidt	Financial Mathematics, Machine Learning
Prof. Dr. Wolfgang Soergel	Algebra and Representation Theory
Prof. Dr. Guofang Wang	Partial Differential Equations, Calculus of Variations

On https://www.math.uni-freiburg.de/forschung/index.html, the research areas are described in more detail.

# Offers from EUCOR Partner Universities

As part of the EUCOR cooperation, you can attend courses at the partner universities University of Basel, Karlsruhe Institute of Technology, Université Haute-Alsace in Mulhouse, and the Université de Strasbourg. The procedure is explained in detail on this information page.

In particular, Basel and Strasbourg offer interesting additions to our lecture programme at the master's level. Credits can be recognized within the framework of the respective examination regulations, especially in the elective (required) area of the B.Sc. and M.Sc. programmes. Please discuss possible credits in advance with the programme coordination!

The costs for travel by train, bus, and tram can be subsidized by EUCOR.

# Basel

**Institute:** The Department of Mathematics and Computer Science at the University of Basel offers eight research groups in mathematics: Algebraic Geometry, Number Theory, Analysis, Numerics, Computational Mathematics, Probability Theory, Mathematical Physics, and Statistical Science.

Lecture Offerings: The pages with the lecture offerings for the Bachelor and the lecture offerings for the Master seem to correspond most closely to our mathematics lecture directory. The general lecture directory of the university can be found here: https://vorlesungsverzeichnis.unibas.ch/de/semester-planung

**Dates:** In Basel, the autumn semester begins in mid-September and ends at the end of December, the spring semester runs from mid-February to the end of May.

**Travel:** The University of Basel is best reached by train: The train ride to the Badischer Bahnhof takes about 45-60 minutes by local transport, 30 minutes by ICE. Then take tram 6 towards *Allschwil Dorf* to the *Schifflände* stop (about 10 minutes).

# Strasbourg

**Institute:** In Strasbourg, there is a large *Institut de recherche mathématique avancée* (IRMA), which is divided into seven *équipes*: Analyse; Arithmétique et géométrie algébrique; Algèbre, représentations, topologie; Géométrie; Modélisation et contrôle; Probabilités und Statistique. Seminars and working groups (*groupes de travail*) are announced on the institute's website.

Lecture Offerings: Participation of Freiburg students in the offers of the second year of the Master M2 is highly welcome. Depending on prior knowledge, the lectures are suitable for our students from the 3rd year of study onwards. The lecture language is a priori French, but a switch to English is gladly possible upon request, please arrange in advance. In Strasbourg, a different focus topic is offered in the M2 annually, in 2024/25 it is: Algebra and Topology.

General lecture directories typically do not exist in France.

**Dates:** In France, the *1st semester* runs from early September to late December and the *2nd semester* from late January to mid-May. A more precise schedule will only be available in September. The timetables are flexible; they can usually accommodate the needs of Freiburg students.

**Travel:** The Université de Strasbourg is best reached by car (about one hour). Alternatively, there is a very affordable connection with Flixbus to the *Place de l'Étoile*. The train ride to the main station in Strasbourg takes about 1h40 by local transport, 1h10 by ICE. Then take tram line C towards Neuhof, Rodolphe Reuss to the Universités stop.

For further information and organizational assistance, please contact:

- in Freiburg: Prof. Dr. Annette Huber-Klawitter annette.huber@math.uni-freiburg.de
- in Strasbourg: Prof. Carlo Gasbarri, Coordinator of the M2 gasbarri@math.unistra.fr or the respective course coordinators.

# 1a. Mandatory Lectures of the Study Programmes

# Analysis I

Michael Růžička, Assistant: Alexei Gazca Lecture: Tue, Wed, 8–10 h, HS Rundbau, Albertstr. 21 Tutorial: 2 hours, various dates in German

#### Content:

Analysis I is one of the two basic lectures in the mathematics course. It deals with concepts based on the notion of limit. The central topics are: induction, real and complex numbers, convergence of sequences and series, completeness, exponential function and trigonometric functions, continuity, derivation of functions of one variable and regulated integrals.

#### Literature:

To be announced in the lecture.

#### **Prerequisites:**

Required: High school mathematics. Attendance of the preliminary course (for students in mathematics) is recommended.

	• Analysis (2HfB21, BSc21, MEH21, MEB21) – 18 ECTS	<ul> <li>Analysis I (BScInfo19, BScPhys20) –</li> <li>9 ECTS</li> </ul>
PL: Oral examination (duration approx. 30 minutes) on Analysis I and II at the end of the module. (Passing the written exam on Analysis I and the exercises on Analysis II are admission requirements).	×	
SL: Passing the final sit-in exam (duration 1 to 3 hours).	×	×
SL: Regular participation in the exercise sessions (as defined in the examination regulations).	×	×
SL: At least $50\%$ of the points that can be achieved for the exercises.	×	×
SL: Students must be able to present their submitted solutions in the exercise sessions at the tutor's request.	×	×
	1	2

① The course only covers part of the module. ECTS points are only credited for the complete module.

# Linear Algebra I

Stefan Kebekus, Assistant: Marius Amann Lecture: Mon, Thu, 8–10 h, HS Rundbau, Albertstr. 21 Tutorial: 2 hours, various dates in German

#### Content:

Linear Algebra I is one of the two introductory lectures in the mathematics degree program that form the basis for further courses. Topics covered include: fundamental concepts (in particular fundamental concepts of set theory and equivalence relations), groups, fields, vector spaces over arbitrary fields, basis and dimension, linear mappings and transformation matrix, matrix calculus, linear systems of equations, Gaussian elimination, linear forms, dual space, quotient vector spaces and homomorphism theorem, determinant, eigenvalues, polynomials, characteristic polynomial, diagonalizability, affine spaces. The background to the mathematical content is explained in terms of ideas and the history of mathematics.

#### Literature:

To be announced in the lecture.

#### **Prerequisites:**

Required: High school mathematics. Attendance of the preliminary course (for students in mathematics) is recommended.

	• Linear Algebra (2HfB21, BSc21, MEH21) – 18 ECTS	• Linear Algebra (MEB21) – 15 ECTS	• Linear Algebra I (BScInfo19, BScPhys20) – 9 ECTS
PL: Oral examination (duration approx. 30 minutes) on Linear Algebra I and II at the end of the module. (Passing the written exam on Linear Algebra I and the exercises on Linear Algebra II are admission requirements).	×		
SL: Passing the final sit-in exam (duration 1 to 3 hours).	×	×	×
SL: Regular participation in the exercise sessions (as defined in the examination regula- tions).	×	×	×
SL: At least 50% of the total points that can be achieved for the exercises and the quizzes.	x	×	×
	1	2	3

① ② The course only covers part of the module. ECTS points are only credited for the complete module.

## Numerics I

Sören Bartels, Assistant: Tatjana Schreiber Lecture: Wed, 14–16 h, HS Weismann-Haus, Albertstr. 21a Tutorial: 2 hours, every other week, various dates

#### in German

#### Content:

Numerics is a sub-discipline of mathematics that deals with the practical solution of mathematical problems. As a rule, problems are not solved exactly but approximately, for which a sensible compromise between accuracy and computational effort must be found. The first part of the two-semester course focuses on questions of linear algebra such as solving linear systems of equations and determining the eigenvalues of a matrix. Attendance at the accompanying practical exercises (*Praktische Übung zur Numerik*) is recommended. These take place every 14 days, alternating with the lecture's tutorial.

#### Literature:

- S. Bartels: Numerik 3x9. Springer, 2016.
- R. Plato: Numerische Mathematik kompakt. Vieweg, 2006.
- R. Schaback, H. Wendland: Numerische Mathematik. Springer, 2004.
- J. Stoer, R. Burlisch: Numerische Mathematik I, II. Springer, 2007, 2005.
- G. Hämmerlin, K.-H. Hoffmann: Numerische Mathematik. Springer, 1990.
- P. Deuflhard, A. Hohmann, F. Bornemann: Numerische Mathematik I, II. DeGruyter, 2003.

#### **Prerequisites:**

Required: Linear Algebra I Recommended: Linear Algebra II and Analysis I (required for Numerics II)

#### **Remarks**:

A computer exercise (*Praktische Übung zur Numerik*) is offered to accompany the lecture.

	<ul> <li>Numerics (BSc21) - 4.5 ECTS</li> <li>Numerics (2HfB21, MEH21) - 4.5 ECTS</li> </ul>	• Numerics I (MEB21) – 5 ECTS
PL: Sit-in exam on Numerics I and II at the end of the summer term (duration: 1 to 3 hours)	×	
PL: Oral examination (duration approx. 30 minutes)		x
SL: At least $50\%$ of the points that can be achieved for the exercises.	×	×
	1	2

① The requirements apply for both semesters of the module separately.

0 The course only covers part of the module. ECTS points are only credited for the complete module.

# **Elementary Probability Theory I**

Angelika Rohde, Assistant: Johannes Brutsche Lecture: Fri, 10–12 h, HS Weismann-Haus, Albertstr. 21a Tutorial: 2 hours, every other week, various dates 07.03., 9–10 h in German

#### Content:

Stochastic is, to put it loosely, the "mathematics of chance", about which—possibly contrary to first impressions many precise and not at all random statements can be formulated and proven. The aim of the lecture is to give an introduction to stochastic modeling, to explain some basic concepts and results of Stochastic and to illustrate them with examples. It is also intended as a motivating preparation for the lecture "Probability Theory" in the summer semester, especially for students in the B.Sc. in Mathematics. Topics covered include: Discrete and continuous random variables, probability spaces and measures, combinatorics, expected value, variance, correlation, generating functions, conditional probability, independence, weak law of large numbers, central limit theorem. The lecture Elementary Probability Theory II in the summer semester will mainly be devoted to statistical topics. If you are interested in a practical, computer-supported implementation of individual lecture contents, participation in the regularly offered practical excercise "Praktischen Übung Stochastik" is also recommended (in parallel or subsequently).

#### Literature:

- L. Dümbgen: Stochastik für Informatiker, Springer, 2003.
- H.-O. Georgii: Stochastik: Einführung in die Wahrscheinlichkeitstheorie und Statistik (5. Auflage), De Gruyter, 2015.
- N. Henze: Stochastik für Einsteiger, (13. Auflage), Springer Spektrum, 2021.
- N. Henze: Stochastik: Eine Einführung mit Grundzügen der Maßtheorie, Springer Spektrum, 2019.
- G. Kersting, A. Wakolbinger: *Elementare Stochastik* (2. Auflage), Birkhäuser, 2010.

#### Prerequisites:

Required: Linear Algebra I, Analysis I and II. Note that Linear Algebra I can be attended in parallel.

	• Elementary Probabilty Theory (2HfB21, MEH21) – 4.5 ECTS	• Elementary Probability Theory I (BSc21, MEB21, MEdual24) – 5 ECTS
PL: Sit-in exam on Elementary Probability Theory I and II at the end of the summer term (duration: 2 to 4 hours)	×	
PL: Sit-in exam on Elementary Probability Theory I at the end of the winter term (duration: 1 to 2 hours)		×
SL: At least 50% of the points that can be achieved for the exercises.	×	×
		-

0 The requirements apply for both semesters of the module separately.

① The course only covers part of the module. ECTS points are only credited for the complete module.

O In the dual M.Ed. programme only in the subject combination with physics.

# Further Chapters in Analysis

Nadine Große, Assistant: Jonah Reuß Lecture: Wed, 8–10 h, HS Weismann-Haus, Albertstr. 21a Tutorial: 2 hours, various dates

#### Content:

Multiple integration: Jordan content in  $\mathbb{R}^n$ , Fubini's theorem, transformation theorem, divergence and rotation of vector fields, path and surface integrals in  $\mathbb{R}^3$ , Gauss' theorem, Stokes' theorem. Complex analysis: Introduction to the theory of holomorphic functions, Cauchy's integral theorem, Cauchy's integral formula and applications.

#### Literature:

- K. Königsberger: Analysis 2, 5. Auflage., Springer, 2004.
- W. Walter: Analysis 2, 5. Auflage, Springer, 2002.
- E. Freitag, R. Busam: Funktionentheorie I, 4. Auflage, Springer, 2006.
- R. Remmert, G. Schumacher: Funktionentheorie 1. 5. Auflage, Springer, 2002.

#### **Prerequisites:**

Required: Analysis I and II, Linear Algebra I and II

in German

	<ul> <li>Further Chapters in Analysis (MEd18, MEH21, MEdual24) - 5 ECTS</li> </ul>
PL: Sit-in exam (duration: 1 to 3 hours)	×
SL: At least 50% of the points that can be achieved for the exercises.	×
	0

① In the dual M.Ed. programme only in the subject combination with computer science.

# **Basics in Applied Mathematics**

Moritz Diehl, Patrick Dondl, Angelika Rohde, Assistant: Ben Deitmar, Coffi Aristide Hounkpe Lecture: Tue, Thu, 8–10 h, HS II, Albertstr. 23b Tutorial: 2 hours, date to be determined Computer exercise: 2 hours, date to be determined

in English

#### Content:

This course provides an introduction into the basic concepts, notions, definitions and results in probability theory, numerics and optimization, accompanied with programming projects in Python. Besides deepen mathematical skills in principle, the course lays the foundation of further classes in these three areas.

#### Literature:

Lecture notes will be provided.

#### **Prerequisites:**

None that go beyond admission to the degree programme.

	<ul> <li>Basics in Applied Mathematics (MScData24) - 12 ECTS</li> </ul>
SL: In each of the three parts (stochastics, numerics, optimisation) at least 50% of the points that can be achieved for the exercises.	×
SL: Successful completion of the programming tasks.	×
SL: Passing a final oral test.	×

# 1b. Advanced 4-hour Lectures

# Algebra and Number Theory

Wolfgang Soergel, Assistant: Damian Sercombe Lecture: Tue, Thu, 10–12 h, HS Weismann-Haus, Albertstr. 21a Tutorial: 2 hours, various dates

#### Content:

This lecture continues the linear algebra courses. It treats groups, rings, fields and applications in the number theory and geometry. The highlights of the lecture are the classification of finite fields, the impossibility of the trisection of angles with circle and ruler, the non-existence of a solution formula for the general equations of fifth degree and the quadratic reciprocity law.

#### Literature:

- Michael Artin: Algebra, Birkhäuser 1998.
- Siegfried Bosch: Algebra (8. Auflage.), Springer Spektrum 2013.
- Serge Lang: Algebra (3. Auflage.), Springer 2002.
- Wolfgang Soergel: Script Algebra und Zahlentheorie

#### **Prerequisites:**

Required: Linear Algebra I and II

in German

	• Algebra and Number Theory (2HfB21, MEH21) – 9 ECTS	• Compulsory Elective in Mathematics (BSc21) – 9 ECTS	• Introduction to Algebra and Number Theory (MEB21) – 5 ECTS	• Algebra and Number Theory (MEdual24) – 9 ECTS	• Pure Mathematics (MSc14) – 11 ECTS	<ul> <li>Elective (MSc14) - 9 ECTS</li> <li>Elective (MScData24) - 9 ECTS</li> </ul>
PL: Sit-in exam (duration: 1 to 3 hours)	×	×				
PL: Oral examination (duration approx. 30 minutes)				×	×	
PL: Oral examination on the first part of the lecture until Christmas (duration: approx. 30 minutes)			×			
SL: Passing the final sit-in exam (duration 1 to 3 hours).					×	×
SL: At least 50% of the points that can be achieved for the exercises.	×	×	×	×	×	×
	1	2	3	4	5	6

<sup>②</sup> Counts as one of the three modules 'Lecture with Exercise A' to 'Lecture with Exercise C', if needed, and covers the condition that one must belong to pure mathematics.

③ Only the exercise sheets that can be completed by Christmas count.

# Algebraic Number Theory

Abhishek Oswal, Assistant: Andreas Demleitner Lecture: Tue, Thu, 12–14 h, HS II, Albertstr. 23b Tutorial: 2 hours, date to be determined

#### Content:

Short description of topics: Number fields, Prime decomposition in Dedekind domains, Ideal class groups, Unit groups, Dirichlet's unit theorem, local fields, valuations, decomposition and inertia groups, introduction to class field theory.

#### Literature:

Jürgen Neukirch: Algebraic Number Theory, Springer, 1999.

#### **Prerequisites:**

Required: Algebra and Number Theory

in English

	• Elective (Option Area) (2HfB21) – 9 ECTS	• Compulsory Elective in Mathematics (BSc21) – 9 ECTS	<ul> <li>Mathematical Concentration (MEd18, MEH21) - 9 ECTS</li> <li>Pure Mathematics (MSc14) - 11 ECTS</li> <li>Mathematics (MSc14) - 11 ECTS</li> </ul>	Concentration Module (MSc14) –     10.5 ECTS	<ul> <li>Elective (MSc14) - 9 ECTS</li> <li>Elective (MScData24) - 9 ECTS</li> </ul>
PL: Oral examination (duration approx. 30 minutes)		×	×		
PL: Oral examination on all parts of the module (duration approx. 45 minutes)				×	
SL: Passing a final oral test.	×				×
SL: At least 50% of the points that can be achieved for the exercises.	×	×	×	×	×
SL: At least one presentation of exercise solutions in the exercise sessions.	×	×	×	×	×
	1	2	3	4	5

 $\oplus$  Can be used if the option 'Individuelle Studiengestaltung' is chosen.

<sup>②</sup> Counts as one of the three modules 'Lecture with Exercise A' to 'Lecture with Exercise C', if needed, and covers the condition that one must belong to pure mathematics.

④ The course only covers part of the module. ECTS points are only credited for the complete module.

(1) The composition of the concentration module must be agreed with the examiner. Not all combinations are allowed.

# Analysis III

Patrick Dondl, Assistant: Oliver Suchanin GermanLecture: Mon, 12–14 h, HS Rundbau, Albertstr. 21, Wed, 10–12 h, HS Weismann-Haus, Albertstr. 21ain GermanTutorial: 2 hours, various dates19.02., 10: 15–11: 45 h, HS Rundbau, Albertstr. 21

#### Content:

Lebesgue measure and measure theory, Lebesgue integral on measure spaces and Fubini's theorem, Fourier series and Fourier transform, Hilbert spaces. Differential forms, their integration and outer derivative. Stokes' theorem and Gauss' theorem.

#### **Prerequisites:**

Required: Analysis I and II, Linear Algebra I

	<ul> <li>Elective (Option Area) (2HfB21) –</li> <li>9 ECTS</li> </ul>	<ul> <li>Analysis III (BSc21) - 9 ECTS</li> <li>Elective in Data (MScData24) - 9 ECTS</li> </ul>	• Mathematical Concentration (MEd18, MEH21) – 9 ECTS
PL: Sit-in exam (duration: 1 to 3 hours)		×	
PL: Oral examination (duration approx. 30 minutes)			×
SL: Passing the final sit-in exam (duration 1 to 3 hours).	×		
SL: At least $50\%$ of the points that can be achieved for the exercises.	×	×	×
	1	2	3

① Can be used if the option 'Individuelle Studiengestaltung' is chosen.

# **Differential Geometry**

Sebastian Goette, Assistant: Mikhail Tëmkin Lecture: Mon, Wed, 14–16 h, HS II, Albertstr. 23b Tutorial: 2 hours, date to be determined in German

#### Content:

Differential geometry, especially Riemannian geometry, deals with the geometric properties of curved spaces. Such spaces also occur in other areas of mathematics and physics, for example in geometric analysis, theoretical mechanics and the general theory of relativity.

#### Literature:

- J. Cheeger, D. G. Ebin, Comparison Theorems in Riemannian Geometry, North-Holland, Amsterdam 1975.
- S. Gallot, D. Hulin, J. Lafontaine, *Riemannian Geometry*, Springer, Berlin-Heidelberg-New York 1987.
- P. Petersen, Riemannian Geometry, Grad. Texts Math. 171, Springer, New York, 2006.

#### **Prerequisites:**

Required: Analysis I–III, Lineare Algebra I and II Recommended: Analysis of Curves and Surfaces ("Kurven und Flächen"), Topology

#### **Remarks:**

A lecture on differential geometry II is expected to be offered in the summer semester 2025.

	• Elective (Option Area) (2HfB21) – 9 ECTS	• Compulsory Elective in Mathematics (BSc21) – 9 ECTS	<ul> <li>Mathematical Concentration (MEd18, MEH21) - 9 ECTS</li> <li>Pure Mathematics (MSc14) - 11 ECTS</li> <li>Mathematics (MSc14) - 11 ECTS</li> </ul>	• Concentration Module (MSc14) – 10.5 ECTS	<ul> <li>Elective (MSc14) - 9 ECTS</li> <li>Elective (MScData24) - 9 ECTS</li> </ul>
PL: Oral examination (duration approx. 20 minutes)		×			
PL: Oral examination (duration approx. 30 minutes)			×		
PL: Oral examination on all parts of the module (duration approx. 45 minutes)				×	
SL: At least on third of the points that can be achieved for the exercises.	×	×	×	×	×
	1	2	3	4	5

① Can be used if the option 'Individuelle Studiengestaltung' is chosen.

<sup>②</sup> Counts as one of the three modules 'Lecture with Exercise A' to 'Lecture with Exercise C', if needed, and covers the condition that one must belong to pure mathematics.

④ The course only covers part of the module. ECTS points are only credited for the complete module.

④ The composition of the concentration module must be agreed with the examiner. Not all combinations are allowed.

# **Introduction to Partial Differential Equations**

Guofang Wang, Assistant: Christine Schmidt Lecture: Mon, Wed, 12–14 h, HS II, Albertstr. 23b Tutorial: 2 hours, date to be determined in German

#### Content:

A large number of different problems from the natural sciences and geometry lead to partial differential equations. Consequently, there can be no talk of an all-encompassing theory. Nevertheless, there is a clear picture for linear equations, which is based on three prototypes: the potential equation  $-\Delta u = f$ , the heat equation  $u_t - \Delta u = f$  and the wave equation  $u_{tt} - \Delta u = f$ , which we will examine in the lecture.

#### Literature:

- E. DiBenedetto: Partial differential equations, Birkhäuser, 2010.
- L. C. Evans: *Partial Differential Equations* (Second Edition), Graduate Studies in Mathematics 19, AMS, 2010.
- Q. Han: A Basic Course in Partial Differential Equations, Graduate Studies in Mathematics 120, AMS, 2011.
- J. Jost: Partial Differential Equations (Third Edition), Springer, 2013.

#### **Prerequisites:**

Required: Analysis III Recommended: Complex Analysis (*Funktionentheorie*)

	• Elective (Option Area) (2HfB21) – 9 ECTS	• Compulsory Elective in Mathematics (BSc21) – 9 ECTS	<ul> <li>Mathematical Concentration (MEd18, MEH21) - 9 ECTS</li> <li>Pure Mathematics (MSc14) - 11 ECTS</li> <li>Mathematics (MSc14) - 11 ECTS</li> </ul>	• Concentration Module (MSc14) – 10.5 ECTS	<ul> <li>Elective (MSc14) - 9 ECTS</li> <li>Elective (MScData24) - 9 ECTS</li> </ul>
PL: Oral examination (duration approx. 30 minutes)		×	×		
PL: Oral examination on all parts of the module (duration approx. 45 minutes)				×	
SL: Passing a final oral test.	×				×
SL: At least 50% of the points that can be achieved for the exercises.	×	×	×	×	×
	1	2	3	4	5

① Can be used if the option 'Individuelle Studiengestaltung' is chosen.

<sup>②</sup> Counts as one of the three modules 'Lecture with Exercise A' to 'Lecture with Exercise C', if needed, and covers the condition that one must belong to pure mathematics.

④ The course only covers part of the module. ECTS points are only credited for the complete module.

④ The composition of the concentration module must be agreed with the examiner. Not all combinations are allowed.

# Introduction to Theory and Numerics of Partial Differential Equations

Sören Bartels, Assistant: Vera Jackisch Lecture: Tue, Thu, 10–12 h, SR 226, Hermann-Herder-Str. 10 Tutorial: 2 hours, date to be determined in English

#### Content:

The aim of this course is to give an introduction into theory of linear partial differential equations and their finite difference as well as finite element approximations. Finite element methods for approximating partial differential equations have reached a high degree of maturity, and are an indispensable tool in science and technology. We provide an introduction to the construction, analysis, and implementation of finite element methods for different model problems. We will address elementary properties of linear partial differential equations along with their basic numerical approximation, the functional-analytical framework for rigorously establishing existence of solutions, and the construction and analysis of basic finite element methods.

#### Literature:

- S. Bartels: Numerical Approximation of Partial Differential Equations, Springer 2016.
- D. Braess: Finite Elemente, Springer 2007.
- S. Brenner, R. Scott: Finite Elements, Springer 2008.
- L. C. Evans: Partial Differential Equations, AMS 2010

#### Prerequisites:

Required: Analysis I and II, Linear Algebra I and II as well as knowledge about higher-dimensional integration (e.g. from Analysis III or Extensions of Analysis)

Recommended: Numerics for differential equations, Functional analysis

	• Elective (Option Area) (2HfB21) – 9 ECTS	• Compulsory Elective in Mathematics (BSc21) – 9 ECTS	<ul> <li>Mathematical Concentration (MEd18, MEH21) - 9 ECTS</li> <li>Applied Mathematics (MSc14) - 11 ECTS</li> <li>Mathematics (MSc14) - 11 ECTS</li> <li>Mathematics (MSc14) - 11 ECTS</li> <li>Advanced Lecture in Numerics (MScData24) - 9 ECTS</li> <li>Elective in Data (MScData24) - 9 ECTS</li> </ul>	Concentration Module (MSc14) –     10.5 ECTS	• Elective (MSc14) – 9 ECTS
PL: Oral examination (duration approx. 30 minutes)		×	×		
PL: Oral examination on all parts of the module (duration approx. 45 minutes)				×	
SL: Passing a final oral test.	×				×
SL: At least 50% of the points that can be achieved for the exercises.	×	×	×	×	×
	1	2	3	4	5

① Can be used if the option 'Individuelle Studiengestaltung' is chosen.

<sup>②</sup> Counts as one of the three modules 'Lecture with Exercise A' to 'Lecture with Exercise C', if needed.

④ The course only covers part of the module. ECTS points are only credited for the complete module.

(1) The composition of the concentration module must be agreed with the examiner. Not all combinations are allowed.

# **Complex Analysis**

David Criens, Assistant: Eric Trébuchon Lecture: Tue, Wed, 16–18 h, HS II, Albertstr. 23b Tutorial: 2 hours, date to be determined 24.02., 14–16 h, HS II, Albertstr. 23b

#### Content:

in German

Complex analysis deals with functions  $f : \mathbb{C} \to \mathbb{C}$ , which map complex numbers to complex numbers. Many concepts of Analysis I can be directly transferred to this case, e.g. the definition of differentiability. One might expect that this would lead to a theory analogous to Analysis I but much more is true: in many respects you get a more elegant and simpler theory. For example, complex differentiability on an open set implies that a function is even infinitely often differentiable, and this is further consistent with analyticity. For real functions, all these notions are different. However, some new ideas are also necessary: For real numbers a, b one integrates for

$$\int_{a}^{b} f(x) \mathrm{d}x$$

over the elements of the interval [a, b] or [b, a]. However, if a, b are complex numbers, it is no longer so clear clear how such an integral is to be calculated. One could, for example, in the complex numbers along the line that connects  $a, b \in \mathbb{C}$ , or along another curve that leads from a to b. Does this lead to a well-defined integral term or does such a curve integral depend on the choice of the curve?

#### **Prerequisites:**

Required: Analysis I+II, Linear Algebra I

	<ul> <li>Elective (Option Area) (2HfB21) –</li> <li>9 ECTS</li> </ul>	• Compulsory Elective in Mathematics (BSc21) – 9 ECTS	• Mathematical Concentration (MEd18, MEH21) – 9 ECTS	• Pure Mathematics (MSc14) – 11 ECTS	<ul> <li>Elective (MSc14) - 9 ECTS</li> <li>Elective (MScData24) - 9 ECTS</li> </ul>
PL: Sit-in exam (duration: 1 to 3 hours)		×			
PL: Oral examination (duration approx. 30 minutes)			×	×	
SL: Passing the final sit-in exam (duration 1 to 3 hours).	×			×	×
SL: At least 50% of the points that can be achieved for the exercises.	×	×	×	×	×
	1	2	3	4	5

0 Can be used if the option 'Individuelle Studiengestaltung' is chosen.

<sup>(2)</sup> Counts as one of the three modules 'Lecture with Exercise A' to 'Lecture with Exercise C', if needed, and covers the condition that one must belong to pure mathematics.

# **Mathematical Statistics**

Ernst August v. Hammerstein, Assistant: Sebastian Stroppel Lecture: Mon, Wed, 14–16 h, SR 404, Ernst-Zermelo-Str. 1 in English

#### Content:

The lecture builds on basic knowledge about Probability Theory. The fundamental problem of statistics is to infer from a sample of observations as precise as possible statements about the data-generating process or the underlying distributions of the data. For this purpose, the most important methods from statistical decision theory such as test and estimation methods are introduced in the lecture.

Key words hereto include Bayes estimators and tests, Neyman-Pearson test theory, maximum likelihood estimators, UMVU estimators, exponential families, linear models. Other topics include ordering principles for reducing the complexity of models (sufficiency and invariance). Statistical methods and procedures are used not only in the natural sciences and medicine, but in almost all areas in which data is collected and analyzed This includes, for example, economics ("econometrics") and the social sciences (especially psychology). However, in the context of this lecture, we will focus less on applications, but—as the name suggests—more on the mathematical justification of the methods.

#### Literature:

- C. Czado, T. Schmidt: *Mathematische Statistik*, Springer, 2011.
- E.L. Lehmann, J.P. Romano: Testing Statistical Hypotheses (Fourth Edition), Springer, 2022.
- E.L. Lehmann, G. Casella: Theory of Point Estimation, Second Edition, Springer, 1998.
- L. Rüschendorf: *Mathematische Statistik*, Springer Spektrum, 2014.
- M. J. Schervish: *Theory of Statistics*, Springer, 1995.
- J. Shao: *Mathematical Statistics*, Springer, 2003.
- H. Witting: Mathematische Statistik I, Teubner, 1985.

#### **Prerequisites:**

Required: Probability Theory (in particular measure theory and conditional probabilities/expectations)

	• Elective (Option Area) (2HfB21) – 9 ECTS	• Compulsory Elective in Mathematics (BSc21) – 9 ECTS	<ul> <li>Applied Mathematics (MSc14) - 11 ECTS</li> <li>Mathematics (MSc14) - 11 ECTS</li> <li>Advanced Lecture in Stochastics (MScData24) - 11 ECTS</li> <li>Elective in Data (MScData24) - 11 ECTS</li> </ul>	Concentration Module (MSc14) –     10.5 ECTS	• Elective (MSc14) – 9 ECTS
PL: Oral examination (duration approx. 30 minutes)		×	×		
PL: Oral examination on all parts of the module (duration approx. 45 minutes)				×	
SL: At least 50% of the points that can be achieved for the exercises.	×	×	×	×	×
SL: At least two presentations of exercise solutions in the exercise sessions.	×	×	×	×	×
	Û	2	3	4	5

① Can be used if the option 'Individuelle Studiengestaltung' is chosen.

<sup>②</sup> Counts as one of the three modules 'Lecture with Exercise A' to 'Lecture with Exercise C', if needed.

④ The course only covers part of the module. ECTS points are only credited for the complete module.

(1) The composition of the concentration module must be agreed with the examiner. Not all combinations are allowed.

## Set Theory – Independence Proofs

Maxwell Levine, Assistant: Hannes Jakob Lecture: Tue, Thu, 12–14 h, SR 404, Ernst-Zermelo-Str. 1 Tutorial: 2 hours, date to be determined in English

#### Content:

How does one prove that something cannot be proved? More precisely, how does one prove that a particular statement does not follow from a particular collection of axioms?

These questions are often asked with respect to the axioms most commonly used by mathematicians: the axioms of Zermelo-Fraenkel set theory, or ZFC for short. In this course, we will develop the conceptual tools needed to understand independence proofs with respect to ZFC. On the way we will develop the theory of ordinal and cardinal numbers, the basics of inner model theory, and the method of forcing. In particular, we will show that Cantor's continuum hypothesis, the statement that  $2^{\aleph_0} = \aleph_1$ , is independent of ZFC.

#### Literature:

- Thomas Jech: Set Theory. The Third Millenium Edition, Springer, 2001.
- Kenneth Kunen: Set Theory: An Introduction to Independence Proofs. North-Holland Pub. Co, 1980.

#### **Prerequisites:**

Required: Mathematical Logic

	• Elective (Option Area) (2HfB21) – 9 ECTS	• Compulsory Elective in Mathematics (BSc21) – 9 ECTS	<ul> <li>Mathematical Concentration (MEd18, MEH21) - 9 ECTS</li> <li>Pure Mathematics (MSc14) - 11 ECTS</li> <li>Mathematics (MSc14) - 11 ECTS</li> </ul>	• Concentration Module (MSc14) – 10.5 ECTS	<ul> <li>Elective (MSc14) - 9 ECTS</li> <li>Elective (MScData24) - 9 ECTS</li> </ul>
PL: Oral examination (duration approx. 30 minutes)		×	×		
PL: Oral examination on all parts of the module (duration approx. 45 minutes)				×	
SL: At least 50% of the points that can be achieved for the exercises.	X	×	×	×	×
	1	2	3	4	5

① Can be used if the option 'Individuelle Studiengestaltung' is chosen.

<sup>②</sup> Counts as one of the three modules 'Lecture with Exercise A' to 'Lecture with Exercise C', if needed, and covers the condition that one must belong to pure mathematics.

④ The course only covers part of the module. ECTS points are only credited for the complete module.

④ The composition of the concentration module must be agreed with the examiner. Not all combinations are allowed.

#### Semi-Algebraic Geometry

Annette Huber-Klawitter, Amador Martín Pizarro, Assistant: Christoph Brackenhofer Lecture: Tue, Thu, 10–12 h, HS II, Albertstr. 23b Tutorial: 2 hours, date to be determined

#### Content:

Semi-algebraic geometry is about properties of subsets of  $\mathbf{R}^n$ , which are given by inequalities of the form

$$f(x_1,\ldots,x_n) \ge 0$$

for polynomials  $f \in \mathbf{R}[X_1, \ldots, X_n]$ .

The theory has many different facets. On the one hand, it can be seen as a version of algebraic geometry over  $\mathbf{R}$  (or even more generally over so-called real closed fields). On the other hand, the properties of these fields are a central tool for the model-theoretic proof of Tarski-Seidenberg's theorem on quantifier elimination in real closed fields. Geometrically, this is interpreted as a projection theorem.

From this theorem, a proof of Hilbert's 17th problem easily follows, which was solved by Artin in 1926.

Is every real polynomial  $P \in \mathbf{R}[x_1, \ldots, x_n]$ , which takes a non-negative value for every n-tuple in  $\mathbf{R}^n$ , a sum of squares of rational functions (i.e., quotients of polynomials)?

In the lecture, we will explore both aspects. Necessary tools from commutative algebra or model theory will be discussed according to the prior knowledge of the audience.

#### Literature:

- A. Prestel: Lecture Notes *Reelle Algebra*.
- L. van den Dries: *Tame topology and o-minimal structures*, London Mathematical Society Lecture Note Series, Cambridge University Press, 1998.
- Jacek Bochnak, Michel Coste & Marie-Françoise Roy: *Real Algebra*, Ergebnisse der Mathematik und ihrer Grenzgebiete 36, Springer Verlag, 1998.

#### **Prerequisites:**

Required: Algebra and Number Theory

Recommended: Knowledge in commutative algebra and algebraic geometry (cf. Kommutative Algebra und Einführung in die algebraische Geometrie), model theory

in German

	• Elective (Option Area) (2HfB21) – 9 ECTS	• Compulsory Elective in Mathematics (BSc21) – 9 ECTS	<ul> <li>Mathematical Concentration (MEd18, MEH21) - 9 ECTS</li> <li>Pure Mathematics (MSc14) - 11 ECTS</li> <li>Mathematics (MSc14) - 11 ECTS</li> </ul>	• Concentration Module (MSc14) – 10.5 ECTS	<ul> <li>Elective (MSc14) - 9 ECTS</li> <li>Elective (MScData24) - 9 ECTS</li> </ul>
PL: Oral examination (duration approx. 30 minutes)		×	×		
PL: Oral examination on all parts of the module (duration approx. 45 minutes)				×	
SL: At least 50% of the points that can be achieved for the exercises.	X	×	×	×	×
	1	2	3	4	5

① Can be used if the option 'Individuelle Studiengestaltung' is chosen.

<sup>②</sup> Counts as one of the three modules 'Lecture with Exercise A' to 'Lecture with Exercise C', if needed, and covers the condition that one must belong to pure mathematics.

④ The course only covers part of the module. ECTS points are only credited for the complete module.

④ The composition of the concentration module must be agreed with the examiner. Not all combinations are allowed.

# Theory and Numerics for Partial Differential Equations – Nonlinear Problems

Sören Bartels, Patrick Dondl Lecture (four hours) Tutorial: 2 hours, date to be determined in English

#### Content:

The lecture addresses the development and analysis of numerical methods for the approximation of certain nonlinear partial differential equations. The considered model problems include harmonic maps into spheres, total-variation regularized minimization problems, and nonlinear bending models. For each of the problems, a suitable finite element discretization is devised, its convergence is analyzed and iterative solution procedures are developed. The lecture is complemented by theoretical and practical lab tutorials in which the results are deepened and experimentally tested.

#### Literature:

- S. Bartels: Numerical methods for nonlinear partial differential equations, Springer, 2015.
- M. Dobrowolski: Angewandte Funktionalanalysis, Springer, 2010.
- L.C. Evans: Partial Differential Equations, 2nd Edition, 2010.

#### **Prerequisites:**

Required: Introduction to Theory and Numerics for PDEs or Introduction to PDEs

#### **Remarks:**

This lecture is offered as a reading course.

	• Elective (Option Area) (2HfB21) – 9 ECTS	• Compulsory Elective in Mathematics (BSc21) – 9 ECTS	<ul> <li>Applied Mathematics (MSc14) - 11 ECTS</li> <li>Mathematics (MSc14) - 11 ECTS</li> <li>Advanced Lecture in Numerics (MScData24) - 11 ECTS</li> <li>Elective in Data (MScData24) - 11 ECTS</li> </ul>	• Concentration Module (MSc14) – 10.5 ECTS	• Elective (MSc14) – 9 ECTS
PL: Oral examination (duration approx. 30 minutes)		×	×		
PL: Oral examination on all parts of the module (duration approx. 45 minutes)				×	
SL: Passing a final oral test.	×				×
SL: At least 50% of the points that can be achieved for the exercises.	×	×	×	×	×
	1	2	3	4	5

① Can be used if the option 'Individuelle Studiengestaltung' is chosen.

<sup>②</sup> Counts as one of the three modules 'Lecture with Exercise A' to 'Lecture with Exercise C', if needed.

 $\circledast$  The course only covers part of the module. ECTS points are only credited for the complete module.

④ The composition of the concentration module must be agreed with the examiner. Not all combinations are allowed.

# Stochastic Processes (Probability Theory II)

Peter Pfaffelhuber, Assistant: Samuel Adeosun Questions session / flipped classroom: Mon, 10–12 h, HS II, Albertstr. 23b Letcure (4 hours): asynchronous videos in English

#### Content:

A stochastic process  $(X_t)_{t \in I}$  is nothing more than a family of random variables, where I is some index set modeling time. Simple examples are random walks, Markov chains, Brownian motion and derived processes. The latter play a particularly important role in the modeling of financial mathematics or questions from the sciences. We will first deal with martingales, which describe fair games. After constructing the Poisson process and Brownian motion, we will focus on properties of Brownian motion. Infinitesimal characteristics of a Markov process are described by generators, which allows a connection to the theory of partial differential equations. Finally, a generalization of the law of large numbers is discussed with the ergodic theorem for stationary stochastic processes. Furthermore, insights are given into a few areas of application, such as biomathematics or random graphs.

#### Literature:

- O. Kallenberg: Foundations of Modern Probability (Third Edition), Springer, 2021.
- A. Klenke: Wahrscheinlichkeitstheorie (4. Auflage), Springer, 2020.
- D. Williams: *Probability with Martingales*, Cambridge University Press, 1991.

#### **Prerequisites:**

Required: Probability Theory I

#### **Remarks:**

The lecture series started with the lecture *Probability Theory I* in summer 2024, and will continue In the summer semester 2025 with the lecture *Probability Theory III (Stochastic Analysis)*.

	• Elective (Option Area) (2HfB21) – 9 ECTS	• Compulsory Elective in Mathematics (BSc21) – 9 ECTS	<ul> <li>Applied Mathematics (MSc14) - 11 ECTS</li> <li>Mathematics (MSc14) - 11 ECTS</li> <li>Advanced Lecture in Stochastics (MScData24) - 11 ECTS</li> <li>Elective in Data (MScData24) - 11 ECTS</li> </ul>	Concentration Module (MSc14) –     10.5 ECTS	• Elective (MSc14) – 9 ECTS
PL: Oral examination (duration approx. 30 minutes)		×	×		
PL: Oral examination on all parts of the module (duration approx. 45 minutes)				×	
SL: Passing a final oral test.	×				×
SL: At least 50% of the points that can be achieved for the exercises.	×	×	×	×	×
SL: At least two presentations of exercise solutions in the exercise sessions.	×	×	×	X	×
	1	2	3	4	5

 $\oplus$  Can be used if the option 'Individuelle Studiengestaltung' is chosen.

<sup>②</sup> Counts as one of the three modules 'Lecture with Exercise A' to 'Lecture with Exercise C', if needed.

(1) The course only covers part of the module. ECTS points are only credited for the complete module.

B The composition of the concentration module must be agreed with the examiner. Not all combinations are allowed.

# Stochastic Integration and Financial Mathematics (Probability Theory III)

Thorsten Schmidt, Assistant: Moritz Ritter Lecture: Mon, Wed, 12–14 h, SR 404, Ernst-Zermelo-Str. 1 Tutorial: 2 hours, date to be determined

in English

#### Content:

This lecture marks the culmination of our series on probability theory, achieving the ultimate goal of this series: the combination of stochastic analysis and financial mathematics—a field that has yielded an amazing wealth of fascinating results since the 1990s. The core is certainly the application of semimartingale theory to financial markets culminating in the fundamental theorem of asset pricing. This results is used everywhere in financial markets for arbitrage-free pricing.

After this we look into modern forms of stochastic analysis covering neural SDEs, signature methods, uncertainty and term structure models. The lecture will conclude with an examination of the latest applications of machine learning in financial markets and the reciprocal influence of stochastic analysis on machine learning.

#### Literature:

Relevant literature will be announced during the course.

#### **Prerequisites:**

Required: Probability Theory II (Stochastic Processes)

	• Elective (Option Area) (2HfB21) – 9 ECTS	• Compulsory Elective in Mathematics (BSc21) – 9 ECTS	<ul> <li>Applied Mathematics (MSc14) - 11 ECTS</li> <li>Mathematics (MSc14) - 11 ECTS</li> <li>Advanced Lecture in Stochastics (MScData24) - 11 ECTS</li> <li>Elective in Data (MScData24) - 11 ECTS</li> </ul>	• Concentration Module (MSc14) – 10.5 ECTS	• Elective (MSc14) – 9 ECTS
PL: Oral examination (duration approx. 30 minutes)		×	×		
PL: Oral examination on all parts of the module (duration approx. 45 minutes)				×	
SL: Passing a final oral test.	×				×
SL: At least 50% of the points that can be achieved for the exercises.	×	×	×	×	×
	1	2	3	4	5

① Can be used if the option 'Individuelle Studiengestaltung' is chosen.

<sup>②</sup> Counts as one of the three modules 'Lecture with Exercise A' to 'Lecture with Exercise C', if needed.

④ The course only covers part of the module. ECTS points are only credited for the complete module.

④ The composition of the concentration module must be agreed with the examiner. Not all combinations are allowed.

## **Reading courses**

Alle Professor:innen und Privatdozen:innen des Mathematischen Instituts Talk/participation possible in German and English

### Content:

In a reading course, the material of a four-hour lecture is studied in supervised self-study. In rare cases, this may take place as part of a course; however, reading courses are not usually listed in the course catalog. If you are interested, please contact a professor or a private lecturer before the start of the course; typically, this will be the supervisor of your Master's thesis, as the reading course ideally serves as preparation for the Master's thesis (both in the M.Sc. and the M.Ed. programs).

The content of the reading course, the specific details, and the coursework requirements will be determined by the supervisor at the beginning of the lecture period. The workload should be equivalent to that of a four-hour lecture with exercises.

	<ul> <li>Reading Course (MEd18, MEH21) - 9 ECTS</li> <li>Mathematics (MSc14) - 11 ECTS</li> </ul>	• Concentration Module (MSc14) – 10.5 ECTS	• Elective (MSc14) – 9 ECTS
PL: Oral examination (duration approx. 30 minutes)	×		
PL: Oral examination on all parts of the module (duration approx. 45 minutes)		×	
SL: Independent reading of lecture notes, articles or book chapters specified by the supervisor and, if required, completion of accompanying exercises. Regular report on the progress with formulation of questions. Up to two presentations to the working group on the material worked on so far, possibly in a research or working group seminar. If the reading course takes place within the framework of a regular course (e.g. seminar or working group seminar): regular participation in this course.	×	×	×
	1	2	3

<sup>②</sup> The course only covers part of the module. ECTS points are only credited for the complete module.

<sup>(2)</sup> The composition of the concentration module must be agreed with the examiner. Not all combinations are allowed.

# 1c. Advanced 2-hour Lectures

## Functions of Bounded Variation and Sets of Finite Perimeter

Xuwen Zhang Lecture: Mon, 14–16 h, SR 127, Ernst-Zermelo-Str. 1 Tutorial: 2 hours, date to be determined in English

#### Content:

We will study functions of bounded variation, which are functions whose weak first partial derivatives are Radon measures. This is essentially the weakest definition of a function to be differentiable in the measure-theoretic sense. After discussing the basic properties of them, we move on to the study of sets of finite perimeter, which are Lebesgue measurable sets in the Euclidean space whose indicator functions are BV functions. Sets of finite perimeter are fundamental in the modern Calculus of Variations as they generalize in a natural measure-theoretic way the notion of sets with regular boundaries and possess nice compactness, thus appearing in many Geometric Variational problems. If time permits, we will discuss the (capillary) sessile drop problem as one important application.

#### Literature:

- Evans, Lawrence C. and Gariepy, Ronald F. Measure theory and fine properties of functions. CRC Press, Boca Raton, FL, 2015.
- Maggi, Francesco. Sets of finite perimeter and geometric variational problems: an introduction to Geometric Measure Theory. No. 135. Cambridge University Press, 2012.

#### **Prerequisites:**

Required: Basic knowledge in measure theory and analysis is required.

	<ul> <li>Elective (Option Area) (2HfB21) –</li> <li>6 ECTS</li> </ul>	• Compulsory Elective in Mathematics (BSc21) – 6 ECTS	<ul> <li>Pure Mathematics (MSc14) - 5.5 ECTS</li> <li>Mathematics (MSc14) - 5.5 ECTS</li> </ul>	Concentration Module (MSc14) –     5.25 ECTS	<ul> <li>Elective (MSc14) - 6 ECTS</li> <li>Elective (MScData24) - 6 ECTS</li> </ul>
PL: Oral examination (duration approx. 30 minutes)		×			
PL: Oral examination on all parts of the module (duration approx. 30 minutes)			×		
PL: Oral examination on all parts of the module (duration approx. 45 minutes)				×	
SL: Passing a final oral test.	×				×
SL: At least 50% of the points that can be achieved for the exercises.	X	X	×	×	×
	1	2	3	4	5

 $\oplus$  Can be used if the option 'Individuelle Studiengestaltung' is chosen.

<sup>(2)</sup> The examinations are held by Prof Wang.

③ ④ The course only covers part of the module. ECTS points are only credited for the complete module.

B The composition of the concentration module must be agreed with the examiner. Not all combinations are allowed.

## **Futures and Options**

Eva Lütkebohmert-Holtz, Assistant: Hongyi Shen Lecture: Mon, 10–12 h, HS 3042, KG III

#### Content:

This course covers an introduction to financial markets and products. Besides futures and standard put and call options of European and American type we also discuss interest-rate sensitive instruments such as swaps.

For the valuation of financial derivatives we first introduce financial models in discrete time as the Cox–Ross–Rubinstein model and explain basic principles of risk-neutral valuation. Finally, we will discuss the famous Black–Scholes model which represents a continuous time model for option pricing.

#### Literature:

- D. M. Chance, R. Brooks: An Introduction to Derivatives and Risk Management (10th edition), Cengage, 2016.
- J. C. Hull: Options, Futures, and other Derivatives (11th global edition), Pearson, 2021.
- S. E. Shreve: Stochastic Calculus for Finance I: The Binomial Asset Pricing Model, Springer, 2004.
- R. A. Strong: Derivatives. An Introduction (Second edition), South-Western, 2004.

#### Prerequisites:

Required: Elementary Probability Theory I

#### **Remarks**:

The course is offered for the first year in the Finance profile of the M.Sc. Economics programme as well as for students of M.Sc. and B.Sc. Mathematics, M.Sc. Mathematics in Data and Technology and M.Sc. Volkswirtschaftslehre. In the M.Sc. Mathematics, it can also count as elective in economics for the specialization in financial mathematics. For students who are currently in the B.Sc. Mathematics programme, but plan to continue with this special profile, it is therefore recommended to credit this course for the latter profile and not for B.Sc. Mathematics.

	• Elective (Option Area) (2HfB21) – 6 ECTS	<ul> <li>Compulsory Elective in Mathematics (BSc21) - 6 ECTS</li> <li>Elective in Data (MScData24) - 6 ECTS</li> </ul>	• Supplementary Module in Mathematics (MEd18) – 3 ECTS	<ul> <li>Applied Mathematics (MSc14) -</li> <li>5.5 ECTS</li> <li>Mathematics (MSc14) - 5.5 ECTS</li> </ul>	Concentration Module (MSc14) –     5.25 ECTS	• Elective (MSc14) – 6 ECTS
PL: Sit-in exam (duration: 1 to 3 hours)		×				
PL: Oral examination on all parts of the module (duration approx. 30 minutes)				×		
PL: Oral examination on all parts of the module (duration approx. 45 minutes)					×	
SL: Passing the final sit-in exam (duration 1 to 3 hours).	×		×	×	×	×
	1	2	3	4	5	6

① Can be used if the option 'Individuelle Studiengestaltung' is chosen.

③ The requirements exceed the ECTS points awarded for the module. (There are other options that correspond to the ECTS points awarded for the module).

(1) (5) The course only covers part of the module. ECTS points are only credited for the complete module.

(5) The composition of the concentration module must be agreed with the examiner. Not all combinations are allowed.

© Counts as economics specialisation module in the financial mathematics profile.

# Lie Groups and Symmetric Spaces

Maximilian Stegemeyer Lecture: Thu, 14–16 h, SR 404, Ernst-Zermelo-Str. 1 Exercise session: Tue, 8–10 h, SR 127, Ernst-Zermelo-Str. 1 in English

#### Content:

Lie groups and operations of Lie groups play a central role in geometry and topology. They can be used to describe continuous symmetries, one of the most important concepts of mathematics and physics. Exploiting symmetries, e.g. when describing homogeneous spaces, makes it easier to solve many specific problems and often provides a deeper insight into the structures examined. In addition, the geometry and topology of Lie groups and homogeneous spaces is of great interest.

In this lecture, we start with introducing the basic theory of Lie groups and Lie algebras, especially with insights into the structure theory of Lie algebras. In the second part we will look at homogeneous spaces with a special focus on Riemannian symmetric spaces. The latter form an important class of examples of Riemannian manifolds. In addition to the Lie-theoretical aspects, a special focus will always be on the homogeneous Riemannian metrics of the respective spaces.

#### Literature:

- S. Helgason. Differential geometry and symmetric spaces. American Mathematical Soc., 2001.
- J.M. Lee: Smooth manifolds. Springer New York, 2012.
- B. O'Neill: Semi-Riemannian geometry with applications to relativity. Academic press, 1983.
- W. Ziller: Lie Groups. Representation Theory and Symmetric Spaces. Lecture Notes, 2010.

#### Prerequisites:

Required: Differential geometry I

	<ul> <li>Elective (Option Area) (2HfB21) –</li> <li>6 ECTS</li> </ul>	• Compulsory Elective in Mathematics (BSc21) – 6 ECTS	<ul> <li>Pure Mathematics (MSc14) - 5.5 ECTS</li> <li>Mathematics (MSc14) - 5.5 ECTS</li> </ul>	Concentration Module (MSc14) –     5.25 ECTS	<ul> <li>Elective (MSc14) - 6 ECTS</li> <li>Elective (MScData24) - 6 ECTS</li> </ul>
PL: Oral examination (duration approx. 30 minutes)		×			
PL: Oral examination on all parts of the module (duration approx. 30 minutes)			×		
PL: Oral examination on all parts of the module (duration approx. 45 minutes)				×	
SL: Passing a final oral test.	×				×
SL: At least 50% of the points that can be achieved for the exercises.	X	X	×	×	×
	1	2	3	4	5

 $\oplus$  Can be used if the option 'Individuelle Studiengestaltung' is chosen.

<sup>(2)</sup> The examinations are held by Prof Große.

 $\circledast$   $\circledast$  The course only covers part of the module. ECTS points are only credited for the complete module.

B The composition of the concentration module must be agreed with the examiner. Not all combinations are allowed.

### **Markov Chains**

David Criens, Assistant: Dario Kieffer Lecture: Thu, 12–14 h, SR 226, Hermann-Herder-Str. 10 Tutorial: 2 hours, date to be determined in English

#### Content:

The class of Markov chains is an important class of (discrete-time) stochastic processes that are used frequently to model for example the spread of infections, queuing systems or switches of economic scenarios. Their main characteristic is the Markov property, which roughly means that the future depends on the past only through the current state. In this lecture we provide the mathematical foundation of the theory of Markov chains. In particular, we learn about path properties, such as recurrence and transience, state classifications and discuss convergence to the equilibrium. We also study extensions to continuous time. On the way we discuss applications to biology, queuing systems and resource management. If the time allows, we also take a look at Markov chains with random transition probabilities, so-called random walks in random environment, which is a prominent model in the field of random media.

#### Literature:

J. R. Norris: Markov Chains, Cambridge University Press, 1997

#### **Prerequisites:**

Required: Elementary Probability Theory I Recommended: Analysis III, Probability Theory I

	<ul> <li>Elective (Option Area) (2HfB21) –</li> <li>6 ECTS</li> </ul>	<ul> <li>Compulsory Elective in Mathematics (BSc21) - 6 ECTS</li> <li>Elective in Data (MScData24) - 6 ECTS</li> </ul>	• Supplementary Module in Mathematics (MEd18) – 3 ECTS	<ul> <li>Applied Mathematics (MSc14) -</li> <li>5.5 ECTS</li> <li>Mathematics (MSc14) - 5.5 ECTS</li> </ul>	Concentration Module (MSc14) –     5.25 ECTS	• Elective (MSc14) – 6 ECTS
PL: Oral examination (duration approx. 30 minutes)		×				
PL: Oral examination on all parts of the module (duration approx. 30 minutes)				×		
PL: Oral examination on all parts of the module (duration approx. 45 minutes)					×	
SL: Passing a final oral test.	×		×			×
SL: At least 50% of the points that can be achieved for the exercises.	×	×	×	×	×	×
	1	2	3	4	5	6

0 Can be used if the option 'Individuelle Studiengestaltung' is chosen.

③ The requirements exceed the ECTS points awarded for the module. (There are other options that correspond to the ECTS points awarded for the module).

(1) (5) The course only covers part of the module. ECTS points are only credited for the complete module.

(5) The composition of the concentration module must be agreed with the examiner. Not all combinations are allowed.

## **Measure Theory**

Peter Pfaffelhuber, Assistant: Samuel Adeosun Exercise session: Wed, 10–12 h, HS II, Albertstr. 23b in English

#### Content:

Measure Theory is the foundation of advanced probability theory. In this course, we build on knowledge in analysis and provide all necessary results for later classes in statistics, probabilistic machine learning and stochastic processes. It contains set systems, constructions of measures using outer measures, the integral, and product measures.

#### Literature:

- H. Bauer. Measure and Integration Theory. deGruyter, 2001.
- V. Bogatchev. Measure Theory. Springer, 2007.
- O. Kallenberg. Foundations of Modern Probability Theory. Springer, 2021.

#### **Prerequisites:**

Required: Basic courses in analysis, and an understanding of mathematical proofs.

#### **Remarks:**

This course is based on self-study of the material, but comes with graded exercises.

	• Elective in Data (MScData24) - 6 ECTS
PL: Oral examination (duration approx. 30 minutes)	×
SL: At least 50% of the points that can be achieved for the exercises.	×
SL: At least two presentations of exercise solutions in the exercise sessions.	×

# Mathematical Physics

Wolfgang Soergel Lecture: Tue, 16–18 h, SR 403, Ernst-Zermelo-Str. 1 in German

#### Content:

Introduction to classic mechanics from the point of view of mathematics. We start with the mathematical modelling of space and time. Then we discuss Newton's equations of movement, physical systems with compulsory conditions, the D'Alembert principle, the Hamilton formalism and its derivation from the Newton's equations and applications of Hamilton formalism.

#### Literature:

- V.Arnold: Mathematical Methods of Classical Mechanics
- Abraham-Marsden: Foundations of Mechanics

#### **Prerequisites:**

Required: Analysis III

	<ul> <li>Elective (Option Area) (2HfB21) - 3 ECTS</li> <li>Elective (BSc21) - 3 ECTS</li> <li>Supplementary Module in Mathematics (MEd18) - 3 ECTS</li> </ul>	• Compulsory Elective in Mathematics (BSc21) – 6 ECTS
PL: Presentation (duration: 45 to 90 minutes)		×
SL: Passing a final oral test.	×	

## Numerical Approximation of Stochastic Differential Equations

Diyora Salimova, Assistant: Ilkhom Mukhammadiev Lecture: Tue, Fri, 12–14 h, SR 226, Hermann-Herder-Str. 10 Tutorial: 2 hours, date to be determined Computer exercise: 2 hours, date to be determined in English

#### Content:

The aim of this course is to enable the students to carry out simulations and their mathematical analysis for stochastic models originating from applications such as mathematical finance and physics. For this, the course teaches a decent knowledge on stochastic differential equations (SDEs) and their solutions. Furthermore, different numerical methods for SDEs, their underlying ideas, convergence properties, and implementation issues are studied.

#### Literature:

- P. E. Kloeden and E. Platen: Numerical Solution of Stochastic Differential Equations. Springer-Verlag, Berlin, 1992.
- Bernt Oksendal: Stochastic Differential Equations, Springer, 2010.

#### **Prerequisites:**

Required: Probability and measure theory, basic numerical analysis and basics of MATLAB programming.

	<ul> <li>Elective (Option Area) (2HfB21) –</li> <li>6 ECTS</li> </ul>	<ul> <li>Compulsory Elective in Mathematics (BSc21) - 6 ECTS</li> <li>Elective in Data (MScData24) - 6 ECTS</li> </ul>	• Supplementary Module in Mathematics (MEd18) – 3 ECTS	<ul> <li>Applied Mathematics (MSc14) -</li> <li>5.5 ECTS</li> <li>Mathematics (MSc14) - 5.5 ECTS</li> </ul>	Concentration Module (MSc14) –     5.25 ECTS	• Elective (MSc14) – 6 ECTS
PL: Oral examination (duration approx. 30 minutes)		×				
PL: Oral examination on all parts of the module (duration approx. 30 minutes)				×		
PL: Oral examination on all parts of the module (duration approx. 45 minutes)					×	
SL: Passing a final oral test.	×		×			×
SL: At least 50% of the points that can be achieved for the exercises.	×	×	×	×	×	×
	1	2	3	4	5	6

0 Can be used if the option 'Individuelle Studiengestaltung' is chosen.

③ The requirements exceed the ECTS points awarded for the module. (There are other options that correspond to the ECTS points awarded for the module).

(1) (5) The course only covers part of the module. ECTS points are only credited for the complete module.

(5) The composition of the concentration module must be agreed with the examiner. Not all combinations are allowed.

# Numerical Optimal Control

Moritz Diehl, Assistant: Florian Messerer Tutorial / flipped classroom: Tue, 14–16 h, HS II, Albertstr. 23b

#### Content:

The aim of the course is to give an introduction to numerical methods for the solution of optimal control problems in science and engineering. The focus is on both discrete time and continuous time optimal control in continuous state spaces. It is intended for a mixed audience of students from mathematics, engineering and computer science.

The course covers the following topics:

- Introduction to Dynamic Systems and Optimization
- Rehearsal of Newton-type methods and Numerical Optimization
- Algorithmic Differentiation
- Discrete Time Optimal Control
- Dynamic Programming
- Continuous Time Optimal Control
- Numerical Simulation Methods
- Hamilton–Jacobi–Bellmann Equation
- Pontryagin and the Indirect Approach
- Direct Optimal Control
- Real-Time Optimization for Model Predictive Control

The lecture is accompanied by intensive weekly computer exercises offered both in MATLAB and Python (6 ECTS) and an optional project (3 ECTS). The project consists in the formulation and implementation of a self-chosen optimal control problem and numerical solution method, resulting in documented computer code, a project report, and a public presentation.

#### Literature:

- M. Diehl, S. Gros: Numerical Optimal Control, lecture notes.
- J.B. Rawlings, D.Q. Mayne, M. Diehl: *Model Predictive Control*, 2nd Edition, Nobhill Publishing, 2017.
- J. Betts: Practical Methods for Optimal Control and Estimation Using Nonlinear Programming, SIAM, 2010.

#### **Prerequisites:**

Required: Analysis I and II, Linear Algebra I and II Recommended: Numerics I, Ordinary Differential Equations, Numerical Optimization

#### **Remarks:**

Together with the optional programming project, the course counts as a 9 ECTS lecture.

	<ul> <li>Elective (Option Area) (2HfB21) –</li> <li>6 ECTS</li> </ul>	• Compulsory Elective in Mathematics (BSc21) – 6 ECTS	• Supplementary Module in Mathematics (MEd18) – 3 ECTS	<ul> <li>Applied Mathematics (MSc14) -</li> <li>5.5 ECTS</li> <li>Mathematics (MSc14) - 5.5 ECTS</li> </ul>	• Concentration Module (MSc14) – 5.25 ECTS	• Elective (MSc14) – 6 ECTS	<ul> <li>Elective in Data (MScData24) –</li> <li>6 ECTS</li> </ul>
PL: Sit-in exam (duration: 1 to 3 hours)		×					×
PL: Oral examination on all parts of the module (duration approx. 30 minutes)				×			
PL: Oral examination on all parts of the module (duration approx. 45 minutes)					×		
SL: Passing the final sit-in exam (duration 1 to 3 hours).	×		×	×	×	×	
SL: Passing the midterm quiz (at least 80% of the points, any number of attempts within one week).	X	×	×	×	X	×	×
	1	2	3	4	5	6	Ō

① Can be used if the option 'Individuelle Studiengestaltung' is chosen.

(2) (3) (3) (3)) With an additional programming project to be completed, the course counts as a 4-hour lecture with exercises for 9 ECTS (or 11/10.5 in the orally examined modules of the M.Sc. in Mathematics). Assessment for the programming project: Presentation of the project and written report.

<sup>②</sup> Counts in the 9 ECTS variant as one of the three modules 'Lecture with Exercise A' to 'Lecture with Exercise C', if needed.

③ The requirements exceed the ECTS points awarded for the module. (There are other options that correspond to the ECTS points awarded for the module).

(1) (5) The course only covers part of the module. ECTS points are only credited for the complete module.

(5) The composition of the concentration module must be agreed with the examiner. Not all combinations are allowed.

# 2a. Mathematics Education

## Introduction to Mathematics Education

Katharina Böcherer-Linder Mon 10–12 h, SR 226, Hermann-Herder-Str. 10, Fri, 8–10 h, SR 127, Ernst-Zermelo-Str. 1 Fri, 14–16 h, SR 127, Ernst-Zermelo-Str. 1 10.02., 8: 15–10 h

Content:

Mathematics didactic principles and their learning theory foundations and possibilities of teaching implementation (also e.g. with the help of digital media).

Theoretical concepts on central mathematical thinking activities such as concept formation, modeling, problem solving and reasoning.

Mathematics didactic constructs: Barriers to understanding, pre-concepts, basic ideas, specific difficulties with selected mathematical content.

Concepts for dealing with heterogeneity, taking into account subject-specific characteristics particularities (e.g. dyscalculia or mathematical giftedness).

Levels of conceptual rigour and formalization as well as their age-appropriate implementation.

#### **Prerequisites:**

Required: Analysis I, Linear Algebra I

#### **Remarks:**

The course is compulsory in the teaching degree option of the two-main-subject Bachelor's degree program. It is made up of lecture components and parts with exercise and seminar character. The three forms of teaching cannot be not be completely separated from each other. Attendance at the "Didactic Seminar" (approximately fortnightly, tuesday evenings, 19:30) is expected!

This course is only offered in German.

in German

	<ul> <li>(Introduction to) Mathematics Edu (2HfB21, MEH21, MEB21, MEdual2 5 ECTS</li> </ul>
SL: Regular participation in the exercise sessions (as defined in the examination regulations).	×
SL: Written work on the tutorial.	×
SL: Passing the final sit-in exam (duration 1 to 3 hours).	×

## Mathematics Education – Functions and Analysis

Katharina Böcherer-Linder Seminar: Thu, 9–12 h, SR 404, Ernst-Zermelo-Str. 1 in German

#### Content:

Exemplary implementations of the theoretical concepts of central mathematical thought processes such as concept formation, modeling, problem solving and reasoning for the content areas of functions and analysis. Barriers to understanding, pre-concepts, basic ideas, specific difficulties for the content areas of functions and analysis. Fundamental possibilities and limitations of media, in particular of computer-aided mathematical tools mathematical tools and their application for the content areas of functions and analysis. Analysis of individual mathematical learning processes and errors as well as development individual support measures for the content areas of functions and analysis.

#### Literature:

- R. Dankwerts, D. Vogel: Analysis verständlich unterrichten. Heidelberg: Spektrum, 2006.
- G. Greefrath, R. Oldenburg, H.-S. Siller, V. Ulm, H.-G. Weigand: *Didaktik der Analysis. Aspekte und Grund*vorstellungen zentraler Begriffe. Berlin, Heidelberg: Springer 2016.

#### **Prerequisites:**

Required: Introduction to the didactics of mathematics, Knowledge about analysis and numerics

#### **Remarks**:

The two parts can be completed in different semesters semesters, but have a joint final exam, which is offered every semester and written after completing both parts. This course is only offered in German.

	<ul> <li>Mathematics Education for Specific Areas of Mathematics (MEd18, MEH21, MEB21) – 3 ECTS</li> </ul>
PL: Sit-in examination on both parts of the module.	×
SL: Regular participation in the course (as defined in the examination regulations).	×
SL: Seminar presentation with practical and theoretical part.	×
SL: Weekly reading and, if required, homework.	×
	D

0 The course only covers part of the module. ECTS points are only credited for the complete module.

# Mathematics Education – Probability Theory and Algebra

Anika Dreher Seminar: Fri, 9–12 h, SR 226, Hermann-Herder-Str. 10 in German

#### Content:

Exemplary implementations of the theoretical concepts of central mathematical thought processes such as concept formation, modeling, problem solving and reasoning for the content areas of stochastics and algebra. Barriers to understanding, pre-concepts, basic ideas, specific difficulties for the content areas of stochastics and algebra. Basic possibilities and limitations of media, especially computer-based mathematical tools and their mathematical tools and their application for the content areas of stochastics and algebra.

Analysis of individual mathematical learning processes and errors as well as development individual support measures for the content areas of stochastics and algebra.

#### Literature:

- G. Malle: Didaktische Probleme der elementaren Algebra. Braunschweig, Wiesbaden: Vieweg 1993.
- A. Eichler, M. Vogel: Leitidee Daten und Zufall. Von konkreten Beispielen zur Didaktik der Stochastik. Wiesbaden: Vieweg 2009.

#### **Prerequisites:**

Required: Introduction to the didactics of mathematics, knowledge from stochastics and algebra.

#### Remarks:

The two parts can be completed in different semesters semesters, but have a joint final exam, which is offered every semester and written after completing both parts. This course is only offered in German.

	<ul> <li>Mathematics Education for Specific Areas of Mathematics (MEd18, MEH21, MEB21) - 3 ECTS</li> </ul>
PL: Sit-in examination on both parts of the module.	×
SL: Regular participation in the course (as defined in the examination regulations).	×
SL: Seminar presentation with practical and theoretical part.	×
SL: Weekly reading and, if required, homework.	×
	Û

0 The course only covers part of the module. ECTS points are only credited for the complete module.

### Mathematics education seminar: Media Use in Teaching Mathematics

Jürgen Kury Seminar: Wed, 15–18 h, SR 127, Ernst-Zermelo-Str. 1 in German

#### Content:

The use of teaching media in mathematics lessons wins both at the level of lesson planning and lesson realization in importance. Against the background of constructivist learning theories shows that the reflective use of computer programs, among other things mathematical concept formation in the long term. For example experimenting with computer programs allows mathematical structures to be discovered, without this being overshadowed by individual routine operations (such as term transformation) would be covered up. This has far-reaching consequences for mathematics lessons. For this reason, this seminar aims to provide students the necessary decision-making and action skills to prepare future mathematics teachers for their professional activities. Starting from initial considerations about lesson planning, computers and tablets with regard to their respective didactic potential and tested with learners during a classroom visit. The exemplary systems presented are:

- dynamic geometry Software: Geogebra
- Spreadsheets: Excel
- Apps for Smartphones and tablets

The students should develop teaching sequences, which will then be tested and reflected on with pupils (where this will be possible).

#### **Prerequisites:**

Recommended: Basic courses in mathematics

	<ul> <li>Supplementary Module in Mathematics Education (MEd18, MEH21, MEB21) - 4 ECTS</li> </ul>
SL: Regular participation in the course (as defined in the examination regulations).	×
SL: Active oral participation and preparation and presentation of a teaching sequence.	×

## Mathematics education seminars at Freiburg University of Education

Dozent:innen der PH Freiburg

in German

#### **Content:**

Für das Modul Fachdidaktische Entwicklung" können auch geeignete Veranstaltungen an der PH Freiburg absolviert werden, sofern dort Studienplätze zur Verfügung stehen. Ob Veranstaltungen geeignet sind, sprechen Sie bitte vorab mit Frau Böcherer-Linder ab; ob Studienplätze zur Verfügung stehen, müssen Sie bei Interessen an einer Veranstaltung von den Dozent:inn:en erfragen.

#### **Prerequisites:**

Für das Modul Fachdidaktische Entwicklung" können auch geeignete Veranstaltungen an der PH Freiburg absolviert werden, sofern dort Studienplätze zur Verfügung stehen. Ob Veranstaltungen geeignet sind, sprechen Sie bitte vorab mit Frau Böcherer-Linder ab; ob Studienplätze zur Verfügung stehen, müssen Sie bei Interessen an einer Veranstaltung von den Dozent:inn:en erfragen.

#### **Remarks:**

For the module "Fachdidaktische Entwicklung", suitable suitable courses can also be completed at the PH Freiburg if places are available there. To find out whether courses are suitable are suitable, please discuss in advance with Ms. Böcherer-Linder in advance; you must check whether places are available if you are interested in a course from the lecturers if you are interested in a course.

Most suitable courses will be offered in German.

SL: The requirements are determined by the University of Education and announced in the module $X$		• Supplementary Module in Mathematics Education (MEd18, MEH21, MEB21) - 4 ECTS
handbook there or in the course.	SL: The requirements are determined by the University of Education and announced in the module handbook there or in the course.	×

## Module "Research in Mathematics Education"

Dozent:innen der PH Freiburg, Frank Reinhold 14–16 h Mon, 16–19 h, Raum noch nicht bekannt, PH Freiburg in German

#### Content:

The three related courses of the module prepare students for an empirical Master thesis in mathematics didactics. The course is jointly designed by all professors at the PH with mathematics didactics research projects at secondary levels 1 and 2 and is carried out by one of these researchers. Afterwards, students have the opportunity to start Master thesis with one of these supervisors - usually integrated into larger ongoing research projects.

The first course of the module provides an introduction to strategies of empirical didactic research (research questions, research status, research designs). Students deepen their skills in scientific research and the evaluation of subject-specific didactic research. In the second course (in the last third of the semester) students are introduced to central qualitative and quantitative research methods through concrete work with existing data (interviews, student products, experimental data), students are introduced to central qualitative and quantitative research methods. The third course is an accompanying seminar for the Master thesis.

The main objectives of the module are the ability to receive mathematics didactic research in order to didactic research to clarify questions of practical relevance and to plan an empirical mathematics didactics Master thesis. It will be held as a mixture of seminar, development of research topics in groups and active work with research data. Recommended literature will be depending on the research topics offered within the respective courses. The parts can also be attended in different semesters, for example part 1 in the second Master semester and part 2 in the compact phase of the third Master semester after the practical semester.

#### Remarks:

Three-part module for M.Ed. students who would like to write a didactic Master thesis in mathematics. Participation only after personal registration by the end of the lecture period of the previous semester in the Department of Didactics. Admission capacity is limited.

Pre-registration: If you would like to take part in this module, please register by 30.09.2024 by e-mail to didaktik@math.uni-freiburg.de and to Ralf Erens. This course will only be offered in German.

	<ul> <li>Research in Mathematics Education (MEd18, MEH21, MEB21) – 4 ECTS</li> </ul>
SL: Regular participation in the course (as defined in the examination regulations).	×
SL: In all three parts of the module: Completion of tasks as specified by the lecturer totalling approximately 60 hours.	×

# 2b. Tutorial Module

# Learning by Teaching

#### **Content:**

What characterizes a good tutorial? This question will be discussed in the first workshop and tips and suggestions will be given. Experiences will be shared in the second workshop.

#### **Remarks:**

Prerequisite for participation is a tutoring position for a lecture of the Institute of Mathematics in the current semester (at least one two-hour or two one-hour tutorial groups over the whole semester). Can be used twice in the M.Sc. program in Mathematics. This course is only offered in German.

	• Elective (Option Area) (2HfB21) – 3 ECTS	<ul> <li>Elective (BSc21) - 3 ECTS</li> <li>Elective (MScData24) - 3 ECTS</li> </ul>	• Elective (MSc14) – 3 ECTS
SL: Participation in both workshop parts. Regular participation in the tutor meetings;. Two reciprocal tutorial visits with one (or more) other module participants.	×	×	×
	1	2	3

① ② ③ Prerequisite for participation is a tutor job in the current semester for a lecture of the Mathematical Institute (at least one two-hour or two one-hour exercise groups over the entire semester).

① Can be used if the option 'Individuelle Studiengestaltung' is chosen.

<sup>③</sup> The module can be completed twice in the M.Sc. programme (in different semesters, but possibly in tutorials for the same lecture).

# **2c.** Computer Exercises

# Computer exercises for Introduction to Theory and Numerics of Partial Differential Equations

Sören Bartels, Assistant: Vera Jackisch Computer exercise: 2 hours, date to be determined in English

#### Content:

The computer tutorial accompanies the lecture with programming exercises.

#### **Prerequisites:**

See the lecture – additionally: programming knowledge.

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## Computer exercises in Numerics

Sören Bartels, Assistant: Tatjana Schreiber

in German

#### Content:

In the computer tutorial accompanying the Numerics (first term) lecture the algorithms developed and analyzed in the lecture are put into practice and and tested experimentally. The implementation is carried out in the programming languages Matlab, C++ and Python. Elementary programming knowledge is assumed.

#### **Prerequisites:**

See the lecture Numerics I (which should be attended in parallel or should already have been completed). Additionally: Elementary programming knowledge.

At least 50% of the points that can be achieved for the computer exercises.	× •	• × 2	• × 3
	Computer Exercise (2HfB21, MEH21, MEB21) – 3 ECTS Supplementary Module in Mathematics (MEd18) – 3 ECTS	Elective (Option Area) (2HfB21) – 3 ECTS	Numerics (BSc21) – 12 ECTS

0 3 The requirements apply for both semesters of the course separately.

<sup>②</sup> Can be used if the option 'Individuelle Studiengestaltung' is chosen.

 $\ensuremath{\mathfrak{I}}$  The course only covers part of the module. ECTS points are only credited for the complete module.

**3a. Undergraduate Seminars** 

# **Ordinary Differential Equations and Applications**

Susanne Knies, Ludwig Striet Seminar: Thu, 12–14 h, SR 125, Ernst-Zermelo-Str. 1 Preliminary Meeting 15.07., 13 h, SR 403, Ernst-Zermelo-Str. 1 Preparation meetings: Dates by arrangement

#### Content:

Numerous dynamic processes in the natural sciences can be modeled by ordinary differential equations. In this proseminar we will deal with explicit solution methods for differential equations as well as the application situations (reaction kinetics, predator-prey models, mathematical pendulum, different growth processes, . . . ) which can be described by them.

#### Literature:

Lecture topics and literature can be found on the website!

#### **Prerequisites:**

Analysis I and II, Lineare Algebra I and II

in German

	<ul> <li>Undergraduate Seminar (2HfB21, BSc21, MEH21, MEB21) – 3 ECTS</li> </ul>
PL: Presentation (duration: 45 to 90 minutes)	×
SL: Regular participation in the course (as defined in the examination regulations).	×

## A Ramble through Mathematics

Angelika Rohde, Assistant: Johannes Brutsche Seminar: Wed, 12–14 h, SR 125, Ernst-Zermelo-Str. 1, Fri, 13–15 h, SR 404, Ernst-Zermelo-Str. 1 Preregistration: Preliminary Meeting 16.07., 10: 15 h, Raum 232, Ernst-Zermelo-Str. 1 Preparation meetings: Dates by arrangement

#### Content:

Paul Erdős liked to talk about the *BOOK* in which God keeps the *perfect* proofs of mathematical theorems, according to the famous quote by G. H. Hardy that "there is no permanent place for ugly mathematics" ([1], Preface). In an attempt at a best approximation to this *BOOK*, Aigner and Ziegler have published a large number of sentences with elegant, sophisticated, and sometimes surprising evidence. In this proseminar, a selection of these results will be presented. The spectrum of topics covers all different areas of mathematics, from number theory, geometry, analysis, and combinatorics to graph theory and includes well-known results, such as Littlewood and Offord's lemma, the Dinitz problem, Hilbert's third problem (of his 23 problems presented at the International Congress of Mathematicians in Paris in 1900), the Borsuk conjecture, and many more.

#### Literature:

[1] Martin Aigner, Günter M. Ziegler: Das BUCH der Beweise (5. Auflage), Springer, 2018.

#### **Prerequisites:**

Linear Algebra I and II, Analysis I and II

in German

	<ul> <li>Undergraduate Seminar (2HfB21, BSc21, MEH21, MEB21) – 3 ECTS</li> </ul>
PL: Presentation (duration: 45 to 90 minutes)	×
SL: Regular participation in the course (as defined in the examination regulations).	×

# Undergraduate seminar in Algebra

Wolfgang Soergel, Assistant: Damian Sercombe Seminar: Tue, 14–16 h, SR 127, Ernst-Zermelo-Str. 1 Preregistration: Preparation meetings: Dates by arrangement

#### Content:

In this proseminar we will discuss topics that are found in various textbooks and scripts for basic lectures in linear algebra but which are not part of the standard material. The lectures build on each other only slightly.

#### **Prerequisites:**

Linear Algebra I and II, Analysis I and II.

in German

	<ul> <li>Undergraduate Seminar (2HfB21, BSc21, MEH21, MEB21) - 3 ECTS</li> </ul>
PL: Presentation (duration: 45 to 90 minutes)	×
SL: Regular participation in the course (as defined in the examination regulations).	×

**3b.** Seminars

# Knot Theory

Ernst August v. Hammerstein Seminar Preregistration: Preliminary Meeting 19.07., 16 h, Raum 232, Ernst-Zermelo-Str. 1 Preparation meetings: Dates by arrangement

#### Content:

A knot can be mathematically defined relatively simply as a closed curve in the three-dimensional space  $\mathbb{R}^3$ . From everyday life, one is certainly already familiar with different types of knots, e.g., surgeon's knot, sailor's knots, and many more. The aim of mathematical knot theory is to find characteristic quantities for the description and classification of knots and thus possibly also to be able to decide whether two knots are equivalent, i.e., if they can be transformed into one another through certain operations. Ropes, cords or wires can be used to illustrate knots as well as interlacings. Prospective teachers can use these not only in this seminar, but perhaps also later in the classroom to display different results in a very practical way.

#### Literature:

• C.C. Adams: The Knot Book: An elementary introduction to the mathematical theory of knots, Revised reprint, AMS, 2004.

A pdf file of a preliminary version can be found under https://www.math.cuhk.edu.hk/course\_builder/ 1920/math4900e/Adams--The%20Knot%20Book.pdf.

- G. Burde, H. Zieschang: *Knots* (Second Revides and Extended Edition), de Gruyter, 2003.
- W.B.R. Lickorish: An Introduction to Knot Theory, Springer, 1997.
- C. Livingston: *Knot Theory*. Mathematical Association of America, 1993.

#### **Prerequisites:**

Required: Basic Mathematics courses. Possibly a little knowledge in topology in addition.

#### **Remarks:**

Remaining places can be allocated as proseminar places.

in German

	• Supplementary Module in Mathematics (MEd18) – 3 ECTS	• Undergraduate Seminar (2HfB21, BSc21, MEH21, MEB21) – 3 ECTS	<ul> <li>Elective (Option Area) (2HfB21) –</li> <li>3 ECTS</li> </ul>
PL: Presentation (duration: 45 to 90 minutes)		×	
SL: Presentation (duration: 45 to 90 minutes)	×		×
SL: Regular participation in the course (as defined in the examination regulations).	×	×	×
	1	2	3

③ Can be used if the option 'Individuelle Studiengestaltung' is chosen.

## Machine Learning and Stochastic Analysis

Thorsten Schmidt, Assistant: Moritz Ritter Seminar: Fri, 10–12 h, SR 125, Ernst-Zermelo-Str. 1 Preregistration: Preliminary Meeting 18.10. Preparation meetings: Dates by arrangement Talk/participation possible in German and English

#### Content:

This seminar will focus on theoretical machine learning results, including modern universal approximation theorems, approximation of filtering methods through transformes, application of machine learning methods in financial markets and possibly other related topics. Moreover, we will cover topics in stochastic analysis, like fractional Ito calculus, uncertainty, filtering and optimal transport. You are also invited to suggest related topics.

#### **Prerequisites:**

Required: Basic Probability and either Machine Learning or Probability Theory II (Stochastic Processes).

#### **Remarks:**

If students are interested and have the required theoretic background, seminars can can also be used as a proseminar.

	• Elective (Option Area) (2HfB21) – 6 ECTS	<ul> <li>Mathematical Seminar (BSc21) -</li> <li>6 ECTS</li> <li>Compulsory Elective in Mathematics (BSc21) - 6 ECTS</li> <li>Mathematical Seminar (MSc14) -</li> <li>6 ECTS</li> <li>Mathematical Seminar (MScData24) -</li> <li>6 ECTS</li> <li>Elective in Data (MScData24) -</li> <li>6 ECTS</li> </ul>	• Supplementary Module in Mathematics (MEd18) – 3 ECTS	• Elective (MSc14) – 6 ECTS
PL: Presentation (duration: 45 to 90 minutes)		×		
SL: Presentation (duration: 45 to 90 minutes)	×		×	×
SL: Regular participation in the course (as defined in the examination regulations).	×	×	×	×
	1	2	3	4

0 Can be used if the option 'Individuelle Studiengestaltung' is chosen.

③ The requirements exceed the ECTS points awarded for the module. (There are other options that correspond to the ECTS points awarded for the module).

# Machine-Learning Methods in the Approximation of PDEs

Sören Bartels, Assistant: Tatjana Schreiber Seminar Preregistration: Preliminary Meeting 08.07., 12: 30 h, Office 209, Preparation meetings: Dates by arrangement

Talk/participation possible in German and English

#### Content:

Machine-learning methods have recently been used to approximate solutions of partial differential equations. While in some cases they lead to advantages over classical approaches, their general superiority is widely open. In the seminar we will review the main concepts and recent developments.

#### Literature:

- B. Bohn, J. Garcke, M. Griebel: Algorithmic Mathematics in Machine Learning, SIAM, 2024.
- P. C. Petersen: Neural Network Theory, Lecture Notes, 2022.

#### **Prerequisites:**

Introduction to Theory and Numerics for PDEs

#### **Remarks:**

If students are interested and have the required theoretic background, seminars can can also be used as a proseminar.

	• Elective (Option Area) (2HfB21) – 6 ECTS	<ul> <li>Mathematical Seminar (BSc21) -</li> <li>6 ECTS</li> <li>Compulsory Elective in Mathematics (BSc21) - 6 ECTS</li> <li>Mathematical Seminar (MSc14) -</li> <li>6 ECTS</li> <li>Mathematical Seminar (MScData24) -</li> <li>6 ECTS</li> <li>Elective in Data (MScData24) -</li> <li>6 ECTS</li> </ul>	• Supplementary Module in Mathematics (MEd18) – 3 ECTS	• Elective (MSc14) – 6 ECTS
PL: Presentation (duration: 45 to 90 minutes)		×		
SL: Presentation (duration: 45 to 90 minutes)	×		×	×
SL: Regular participation in the course (as defined in the examination regulations).	×	×	×	×
	1	2	3	4

0 Can be used if the option 'Individuelle Studiengestaltung' is chosen.

③ The requirements exceed the ECTS points awarded for the module. (There are other options that correspond to the ECTS points awarded for the module).

# Medical Data Science

Harald Binder Talk/participation possible in German and English Seminar: Wed, 10–11: 30 h, HS Medizinische Biometrie, 1. OG, Stefan-Meier-Str. 26 Preregistration: Preliminary Meeting 17.07., HS Medizinische Biometrie, 1. OG, Stefan-Meier-Str. 26

#### Content:

To answer complex biomedical questions from large amounts of data, a wide range of analysis tools is often necessary, e.g. deep learning or general machine learning techniques, which is often summarized under the term "Medical Data Science". Statistical approaches play an important rôle as the basis for this. A selection of approaches is to be presented in the seminar lectures that are based on recent original work. The exact thematic orientation is still to be determined.

#### Literature:

Information on introductory literature is given in the preliminary meeting.

#### **Prerequisites:**

Good knowledge of probability theory and mathematical statistics.

#### **Remarks:**

The seminar can serve as basis for a bachelor's or master's thesis.

	• Elective (Option Area) (2HfB21) – 6 ECTS	<ul> <li>Mathematical Seminar (BSc21) -</li> <li>6 ECTS</li> <li>Compulsory Elective in Mathematics (BSc21) - 6 ECTS</li> <li>Mathematical Seminar (MSc14) -</li> <li>6 ECTS</li> <li>Mathematical Seminar (MScData24) -</li> <li>6 ECTS</li> <li>Elective in Data (MScData24) -</li> <li>6 ECTS</li> </ul>	• Supplementary Module in Mathematics (MEd18) – 3 ECTS	• Elective (MSc14) – 6 ECTS
PL: Presentation (duration: 45 to 90 minutes)		×		
SL: Presentation (duration: 45 to 90 minutes)	×		×	×
SL: Regular participation in the course (as defined in the examination regulations).	×	×	×	×
	1	2	3	4

0 Can be used if the option 'Individuelle Studiengestaltung' is chosen.

③ The requirements exceed the ECTS points awarded for the module. (There are other options that correspond to the ECTS points awarded for the module).

# Minimal Surfaces

Guofang Wang, Assistant: Xuwen Zhang Seminar: Wed, 16–18 h, SR 125, Ernst-Zermelo-Str. 1 Preliminary Meeting 17.07., 16 h Preparation meetings: Dates by arrangement

Talk/participation possible in German and English

#### Content:

Minimal surfaces are surfaces in space with a "minimal" area and can be described using holomorphic functions. They occur, for example in the investigation of soap skins and the construction of stable objects (e.g. in architecture). In the investigation of minimal surfaces elegant methods from various mathematical fields such as function theory, calculus of variations, differential geometry and partial differential equations. are applied.

#### Literature:

- R. Osserman: A survey of minimal surfaces, Van Nostrand 1969.
- J.-H. Eschenburg, J. Jost: Differentialgeometrie und Minimalflächen, Springer 2007.
- E. Kuwert: Einführung in die Theorie der Minimalflächen, Skript 1998.
- W. H. Meeks III, J. Pérez: A survey on classical minimal surface theory.
- T. Colding, W. P. Minicozzi: *Minimal Surfaces*, New York University 1999.

#### **Prerequisites:**

Required: Analysis III or knowledge about multidimensional integration and complex analysis. Recommended: Elementary knowledge about differential geometry.

#### **Remarks:**

If students are interested and have the required theoretic background, seminars can can also be used as a proseminar.

	<ul> <li>Elective (Option Area) (2HfB21) –</li> <li>6 ECTS</li> </ul>	<ul> <li>Mathematical Seminar (BSc21) - 6 ECTS</li> <li>Compulsory Elective in Mathematics (BSc21) - 6 ECTS</li> <li>Mathematical Seminar (MSc14) - 6 ECTS</li> </ul>	• Supplementary Module in Mathematics (MEd18) – 3 ECTS	<ul> <li>Elective (MSc14) - 6 ECTS</li> <li>Elective (MScData24) - 6 ECTS</li> </ul>
PL: Presentation (duration: 45 to 90 minutes)		×		
SL: Presentation (duration: 45 to 90 minutes)	×		×	×
SL: Regular participation in the course (as defined in the examination regulations).	×	×	×	×
	1	2	3	4

0 Can be used if the option 'Individuelle Studiengestaltung' is chosen.

<sup>③</sup> The requirements exceed the ECTS points awarded for the module. (There are other options that correspond to the ECTS points awarded for the module).

## Seminar on Algebraic Topology

Sebastian Goette, Assistant: Mikhail Tëmkin Seminar: Tue, 14–16 h, SR 125, Ernst-Zermelo-Str. 1 Preliminary Meeting 16.07., SR 125, Ernst-Zermelo-Str. 1 Preparation meetings: Dates by arrangement Talk/participation possible in German and English

#### Content:

We will discuss advanced topics in algebraic topology. Depending on the interest of the participants we could work on one of the following topics—if you have other topic suggestions, please contact the lecturer.

- The Steenrod algebra. An additional structure on the cohomology modulo p allows finer statements on the existence of continuous mappings, such as the existence of of linearly independent vector fields on spheres. The Wu formulas provide a connection to characteristic classes of manifolds.
- Structured spectra. In order to represent multiplicative (co-)homology functors by spectra, one needs a closed monoidal category of spectra, for example a category of spectra, for example a symmetric or orthogonal spectra. In this context we also get to know model structures better.
- *K*-theory and index theory. Elliptic differential operators on compact manifolds are manifolds are Fredholm operators. Their index can be defined by the theorem of Atiyah–Singer topologically. We prove this theorem using (mainly) topological methods and give some geometric applications.

#### **Prerequisites:**

Algebraic Topology I and II

#### **Remarks**:

Participants take on one or, if interested, several presentations. For the rest of the time, we continue the event as a reading course or lecture.

If students are interested and have the required theoretic background, seminars can also be used as a proseminar. This course can be offered in English.

	<ul> <li>Elective (Option Area) (2HfB21) –</li> <li>6 ECTS</li> </ul>	<ul> <li>Mathematical Seminar (BSc21) - 6 ECTS</li> <li>Compulsory Elective in Mathematics (BSc21) - 6 ECTS</li> <li>Mathematical Seminar (MSc14) - 6 ECTS</li> </ul>	• Supplementary Module in Mathematics (MEd18) – 3 ECTS	<ul> <li>Elective (MSc14) - 6 ECTS</li> <li>Elective (MScData24) - 6 ECTS</li> </ul>
PL: Presentation (duration: 45 to 90 minutes)		×		
SL: Presentation (duration: 45 to 90 minutes)	X		×	×
SL: Regular participation in the course (as defined in the examination regulations).	×	×	×	×
	1	2	3	4

0 Can be used if the option 'Individuelle Studiengestaltung' is chosen.

<sup>③</sup> The requirements exceed the ECTS points awarded for the module. (There are other options that correspond to the ECTS points awarded for the module).

# Theory of Non-Commutative Algebras

Annette Huber-Klawitter, Assistant: Xier Ren Seminar: Fri, 8–10 h, SR 404, Ernst-Zermelo-Str. 1 Preregistration: Preliminary Meeting 15.07., 11 h, SR 318, Ernst-Zermelo-Str. 1 Preparation meetings: Dates by arrangement

#### Content:

In this seminar, we are going to study finite dimensional (unital, possibly non-commutative) algebras over a (commutative) field k. Prototypes are the rings of square matrices over k, finite field extensions, or the algebra  $k^n$  with diagonal multiplication.

We will concentrate on path algebras of finite quivers (German: Köcher). Modules over them are equivalently described as representations of the quiver. Many algebraic properties can be directly understood from properties of the quiver.

#### Literature:

- Frank Anderson, Kent Fuller: Rings and Categories of Modules, GTM 13, Springer, 1992
- Ralf Schiffler: Quiver Representations, CMS Books in Mathematics, Springer, 2014
- Alexander Kirillov Jr.: Quiver Representations, GSM 174, AMS, 2016

#### **Prerequisites:**

Required: Linear Algebra Recommended: Algebra and Number Theory, Commutative Algebra and Introduction to Algebraic Geometry

#### **Remarks:**

Communication with the assistent will be in English. Talks can be given in German or English. If students are interested and have the required theoretic background, seminars can also be used as a proseminar.

Talk/participation possible in German and English

	<ul> <li>Elective (Option Area) (2HfB21) –</li> <li>6 ECTS</li> </ul>	<ul> <li>Mathematical Seminar (BSc21) - 6 ECTS</li> <li>Compulsory Elective in Mathematics (BSc21) - 6 ECTS</li> <li>Mathematical Seminar (MSc14) - 6 ECTS</li> </ul>	• Supplementary Module in Mathematics (MEd18) – 3 ECTS	<ul> <li>Elective (MSc14) - 6 ECTS</li> <li>Elective (MScData24) - 6 ECTS</li> </ul>
PL: Presentation (duration: 45 to 90 minutes)		×		
SL: Presentation (duration: 45 to 90 minutes)	×		x	×
SL: Regular participation in the course (as defined in the examination regulations).	X	×	×	×
	1	2	3	4

0 Can be used if the option 'Individuelle Studiengestaltung' is chosen.

③ The requirements exceed the ECTS points awarded for the module. (There are other options that correspond to the ECTS points awarded for the module).